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Rewrite the expression using rational exponent notation.

1) $(\sqrt[5]{63})^3$

$$63^{3/5}$$

2) $(\sqrt[3]{-25})^4$

$$(-25)^{4/3}$$

3) $(\sqrt[6]{124})^7$

$$124^{7/6}$$

Rewrite the expression using radical notation.

4) $(-57)^{4/3}$

$$\sqrt[3]{-57}^4$$

5) $13^{3/2}$

$$\sqrt{13}^3$$

6) $204^{5/8}$

$$\sqrt[8]{204}^5$$

Evaluate the expression without using a calculator.

7) $(\sqrt[3]{27})^2$

$$\begin{array}{c} 3^2 \\ \textcircled{9} \end{array}$$

8) $(\sqrt[4]{256})^3$

$$\begin{array}{c} 4^3 \\ \textcircled{64} \end{array}$$

9) $(\sqrt[3]{-64})^2$

$$\begin{array}{c} (-4)^2 \\ \textcircled{16} \end{array}$$

10) $36^{3/2}$

$$\begin{array}{c} 6^3 \\ \textcircled{216} \end{array}$$

11) $(25)^{-3/2}$

$$\frac{1}{\textcircled{125}}$$

12) $(16)^{1/4}$

$$\textcircled{2}$$

13) $(-32)^{-3/5}$

$$\frac{1}{\textcircled{-8}}$$

14) $(81)^{-5/2}$

$$\frac{1}{\textcircled{9}^5}$$

15) $(-125)^{-5/3}$

$$\frac{1}{(\textcircled{-5})^5}$$

Simplify the expression using the properties of radicals and rational exponents.

16) $(4^{2/3} \cdot 5^{3/4})^3$

$$\begin{array}{c} 4^2 \cdot 5^{9/4} \\ \textcircled{16} \cdot 5^{9/4} \end{array}$$

17) $(3^{3/2} \cdot 3^3)^{1/3}$

$$\begin{array}{c} \frac{3}{2} \cdot \frac{1}{3} \quad 3 \cdot \frac{1}{3} \\ \textcircled{3}^{1/2} \cdot 3 \\ \textcircled{2}^{3/2} \end{array}$$

18) $((7^{2/3})^{3/5})^3$

$$\begin{array}{c} \frac{2}{3} \cdot \frac{3}{5} \cdot \frac{3}{1} \\ \textcircled{7}^{6/5} \end{array}$$

$$19) \left(\frac{5^2}{5^{7/2}} \right)^{-1/3} = \left(\frac{5^{7/2}}{5^2} \right)^{\frac{1}{3}} = 5^{\frac{1}{2}}$$

$$22) \sqrt{\frac{\sqrt{3}}{\sqrt{7}}} = \left(\frac{3^{\frac{1}{2}}}{7^{\frac{1}{2}}} \right)^{\frac{1}{2}} = \left(\frac{3}{7} \right)^{1/4}$$

Simplify the expression. Assume all variables are positive.

$$25) x^{\sqrt{3}} \cdot x^{-\sqrt{3}} = x^{1+\sqrt{3}}$$

$$26) \sqrt[4]{\frac{x^1}{y^8}} = \frac{x^{1/4}}{y^2}$$

$$28) \frac{x^{4/3} y^{7/6}}{xy} =$$

$$\frac{\frac{4}{3} - \frac{3}{3}}{\frac{7}{6} - \frac{6}{6}} = x^{\frac{1}{3}} y^{\frac{1}{6}}$$

$$31) \left(\frac{(12xy^2)^{1/2}}{(3y^3z)^{1/2}} \right)^{-3}$$

$$\left(\frac{3^{1/2} y^{3/2} z^{1/2}}{12^{1/2} x^{1/2} y} \right)^{-3} = \frac{3^{3/2} y^{9/2} z^{3/2}}{12^{3/2} x^{3/2} y^{3/2}} =$$

$$29) \left(\frac{2x^3 y^{2/3}}{x^{5/3} y^{3/5} z} \right)^3 =$$

$$\frac{2^3 x^9 y^{2/5}}{x^5 y^{9/5} z^3} = \frac{3 x^{4+1/3}}{z^2}$$

$$32) \sqrt[4]{(3x^3)^3 (3x^2)^5} = \sqrt[4]{3^8 x^{19}} =$$

$$= \frac{3^8 x^{19/4} \sqrt[4]{x^3}}{9x^{3/2}} = \left(\frac{3yz}{12x} \right)^{1/2} = \left(\frac{y^2}{4x} \right)^{3/2}$$

Perform the indicated operation. Assume all variables are positive.

$$33) \sqrt{10\sqrt{3} - 6\sqrt{3}}$$

$$= \sqrt{4\sqrt{3}} = 2(3)^{1/4}$$

$$34) 2x^3 \sqrt{x^4 y z^5} + \sqrt[3]{x^7 y z^5}$$

$$= 2x^2 z \sqrt[3]{x y z^2} + x^2 z \sqrt[3]{x y z^2}$$

$$= 3x^2 z \sqrt[3]{x y z^2}$$

$$20) \left(\frac{1^{1/3}}{2^{1/3}} \right)^2 = \frac{1}{2^{2/3}}$$

$$23) \sqrt[5]{(3^3)^2 \cdot (3^4)^2} = \sqrt[5]{3^{14}} = 3^{14/5}$$

$$21) \sqrt[4]{\sqrt{6}} = \left((6^{1/2})^{1/3} \right)^{\frac{1}{4}} = 6^{\frac{1}{24}}$$

$$24) \sqrt[2]{\frac{5}{7} \cdot \frac{1}{\sqrt{5}}} = \frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$$

$$2 \sqrt{\frac{5}{25}} = \frac{2\sqrt{5}}{5}$$

$$27) \left(\frac{x^{1/4}}{x^{1/2}} \right)^{-1} = \frac{x^{\frac{1}{2}-\frac{1}{4}}}{x^{1/4}} = x^{1/4}$$

$$30) \left(\frac{xy^2}{3y^{4/3} z^{1/2}} \right)^{-1/2}$$

$$= \frac{3^{1/2} y^{2/3} z^{1/4}}{x^{1/2} y}$$

$$= \frac{3^{1/2} z^{1/4}}{x^{1/2} y^{1/3}}$$

$$= \frac{(yz)^{3/2}}{(4x)^{3/2}}$$

Solve the equation. Check for extraneous solutions.

$$35) 3(x - 5)^{3/2} - 6 = 18$$

$$\left((x-5)^{3/2} \right)^{2/3} = 8^{2/3}$$

$$x - 5 = 4$$

$$x = 9$$

$$37) \frac{1}{2}(x - 3)^{3/4} + 6 = 9$$

$$\left((x-3)^{3/4} \right)^{4/3} = (6)^{4/3}$$

$$x - 3 = 3 + 6^{4/3}$$

$$39) (\sqrt{x+9})^2 = (3 - \sqrt{x})^2$$

$$x + 9 = 9 - 6\sqrt{x} + x$$

$$x = 0$$

$$41) (\sqrt{x-7})^2 = (\sqrt{x+1} + 2)^2$$

$$x - 7 = x + 1 + 4\sqrt{x+1} + 4$$

$$-12 = 4\sqrt{x+1}$$

$$-3 = \sqrt{x+1}$$

$$9 = x + 1$$

$$\cancel{x} \neq \cancel{-3} \quad \sqrt{1} = \sqrt{9} + 2$$

No sol. $1 \neq 5$

$$36) (5x + 14)^{2/3} + 10 = 6$$
$$\left((5x+14)^{2/3} \right)^{3/2} = (-4)^{3/2}$$

~~0~~

$$38) 2(5x^2 + 10)^{2/3} - 5 = 45$$

$$\left((5x^2 + 10)^{2/3} \right)^{3/2} = 25^{3/2}$$

$$5x^2 + 10 = 125$$

$$5x^2 = 115$$

$$x^2 = 23$$

$$x = \pm \sqrt{23}$$

$$40) (\sqrt{x+3})^2 = (1 + \sqrt{x+1})^2$$

$$x + 3 = 1 + 2\sqrt{x+1} + x + 1$$

$$1 = (2\sqrt{x+1})^2$$

$$1 = 4(x+1)$$

$$1 = 4x + 4$$

$$x = -3/4$$

$$42) (\sqrt{x+8})^2 = (\sqrt{x} + \sqrt{3})^2$$

$$x + 8 = x + 2\sqrt{3x} + 3$$

$$5 = 2\sqrt{3x}$$

$$25 = 4(3x)$$

$$25 = 12x$$

$$\frac{25}{12} = x$$